Accountability of guessability in Remindo

Below you will find an explanation concerning the calculations to negate guessing in Remindo.

Common terms explained

- #distractors: the number of (#) distracters belonging to the question
- Distractor scoring: the amount of points one can score per distractor (under scoring)
- #answers: the amount of different possible answers. E.g. for a multiple-choice question (multiple answers) with 3 distractors that would be 7 = 2^3-1 (blank answer as not permitted)
- Bonus: the amount of bonus points that are rewarded for a fully correct answer, if applicable
- #correct: the amount of correct distractors belonging to the question
- Formula value: The result of e.g. the quotient rule or product rule, if applicable
- minChoice and maxChoice –the minimum and maximum amount of distractors one can choose

Multiple-choice questions

When using multiple-choice questions, various types of questions and scoring are possible.

One answer

The guessability is 1 / #distractors.

The guessing score depends on the type of scoring:

- To “Correct answer” applies bonus * guessability;
- To “distribution of scoring” applies bonus * guessability + distractor scores* guessability.

Example:

- In case of 4 distractors, the guessability is 1 / 4 = 25%.
- In case of 3 distractors with distractor scores 2, 4 and 5, and 2 bonus points, the guessability is 2 * 1/3 +$\sum(2,4,5)$ * 1/3 = 13/3 = ~4.333.

Multiple answers (opt. limited)

First, minChoice and maxChoice are determined based on the types of questions and scoring:

- When using the quotient rule the participant knows the number of distractors to choose, which means minChoice = maxChoice = #correct

- When multiple answers are correct, minChoice = 1 (blank answers are not allowed), maxChoice = #distractors
- When multiple answers are correct (limited), minChoice = 1, maxChoice = the pre-determined limit

#answers depend on minChoice and maxChoice: $\sum_{k=\text{minChoice}}^{\text{maxChoice}} \binom{\#\text{distractors}}{k}$. If minChoice = 1, and maxChoice = #distractors, the following applies for #answers = $2^{\#\text{distractors}}$ -1. The guessability is then 1 / #answers. The guessing score depends on the type of scoring:

- In case of "Correct answer" the following applies: bonus * guessability;
- Under "distribution of scoring", the number of distractors belonging to each answer is looked at. The frequency is multiplied with #distractorscores: bonus & guessability + $\sum_{k=0}^{\text{maxChoice-1}} \binom{\#\text{distractors} - 1}{k}$ * distractor scores * guessability;
- When using the quotient rule and product rule, the correct/incorrect proportion is calculated per valid answer, after which the applicable formula (quotient or product) is applied: bonus * guessability + $\sum(\text{formula value} * \#\text{answers})$ * guessability.

Example:

- In case of 5 distractors, minChoice =2, and maxChoice = 3, the following applies: #answers = $\sum_{k=2}^{3} \binom{5}{k} = 20$, and the guessability is 1 / 20 = 5%.
- In case of 4 distractors, the following applies: #answers = $2^4 - 1 = 15$, and the guessability = 1 / 15 = ~6.667%. With distractor scores 1, 2, 3 and =1, the guessing score is $\sum_{k=0}^{k=4-1} \binom{4-1}{k} * \sum(1,2,3,-1)$ * guessability = ~2.666.

Matching questions (multiple-response questions (some-of-many questions))

Various types of questions and scoring are also possible when using matching questions.

One association

This is effectively one multiple-choice question with one answer, but with distractors distributed over two dimensions: rows and columns. The guessability and guessing score are calculated in a similar way to the multiple-choice question with one answer.

One or more associations per row

This is effectively several multiple-choice questions (opt. with one answer) in one. The guessability and guessing score are calculated in a similar way per row. The guessability of all rows is multiplied; the guessing score is totalled.

Order questions

The first distractor can be put on #distractors spots, the second distractor can be put on #distractors – 1, the third one on #distractors – 2, etc. This results in #answers = #distractors!. The guessability is 1 / #distractors!. The guessing score in case of scoring type “Correct answer” is bonus * guessability.

Example:

- In case of 4 distractors, there are 4! = 24 possible answers. De guessability = 1 / 4! = ~4.167%.

However, it is also possible to apply the quotient rule, which results in a different calculation of the guessing score. The correct/incorrect proportion will be calculated based on the amount of permutation-inversions in one answer, i.e. the amount of sets of two distractors that are put in the wrong order.

Example:

- With distractors A, B, C, D, and E, the given answer E, C, A, B, D contains the following 6 inversions: EC, EA, EB, ED, CA, CB. The other 4 pairs, however, are in the correct order: CD, AB, AD, BD. This results in a correct/incorrect proportion of 4/10 (40%) and 6/10 (60%).

Especially when #distractors are high, #answers is quite high. In order to not have to calculate all possible inversions belong to each permutation, the following generating function by Muir is used $\prod_{j=1}^{n}\left(\frac{1-x^j}{1-x}\right)$ (“On a simple term of a determinant”, Proc. Royal S. Edinborough, 21 (1898-9), 441-477). The coefficients belonging to this function display the amount of permutations with a specific amount of inversions.

Example:

- In case of 4 distractors, the following function applies: $1 * (1 + x) * (1 + x + x^2) * (1 +x + x^2 + x^3) = 1 + 3x + 5x^2 + 6x^3 + 5x^4 + 3x^5 + x^6$.The coefficients are thus: 1, 3, 5, 6, 5, 3, 1. This means that 1 permutation exists without inversions (the assorted permutation), 3 permutations with 1 inversion, 5 with 2 inversions, 6 with 3 inversions, 5 with 4 inversions, 3 with 5 inversions and 1 with 6 inversions (the inversely assorted permutation).

• Afterwards, the quotient rule is applied on each possible correct/incorrect proportion. The result is multiplied by the following coefficients:
$1 * \left(\frac{6/6}{1+0/6}\right) + 3 * \left(\frac{5/6}{1+1/6}\right) + 5 * \left(\frac{4/6}{1+2/6}\right) + 6 * \left(\frac{3/6}{1+3/6}\right) + 5 * \left(\frac{2/6}{1+4/6}\right) + 3 * \left(\frac{1/6}{1+5/6}\right) + 1 * \left(\frac{0/6}{1+6/6}\right)$, which is multiplied by the guessability of 1 / 4!, resulting in a guessing score of ~0.371.

Hotspot questions and Drag & Drop questions

First, the surface area of each circle or hotspot area will be calculated (approximately), both with and without overlap with one or more elevated areas. Afterwards, which answers are possible (not one chosen area, of two or three) is based on minChoice and maxChoice (if one and the same image can be moved three time, the maxChoice is 3), and the chance to give precisely this answer. (Note: one area can only be chosen once. Choosing one area twice does not have to be accounted for.)
Example:

• An image contains three enveloping areas, each covering 20% of the total area, and 25% of each other. This way, 20% * 3 – (2 * 20% * 25%) = 50% of the image is covered (total surface area - 2x overlap).

• Suppose the maximum score applies to the answer area 1 and area 2. The chance of choosing area 1 is 20%, the chance of choosing area 2 afterwards is also 20%. (Normally, the overlap of area 1 and area 2 results in area 2 only has an actual surface area of 15% but as the area can only be chosen once, area 1 is not an option anymore, including the overlap. Opt. other overlapping areas will also <u>not</u> be generated.) The guessability is thus 20% * 20% = 4%.

The guessing score is iterated for all answers, and the score per answer (total of all respective distractors scores) is multiplied with the guessability per answer.

Graphic associate questions

This is effectively a matching question with multiple associations per row. The guessability and guessing score are calculated in a similar way.

Open questions/Text entry questions/Upload questions

The guessability and guessing score for open questions, text entry questions, and upload questions is always 0.

Combined questions

In case of combined questions, the guessability and guessing score is calculated per question that is combined. The guessability is multiplied, and the guessing scores are totalled. In case of a negative guessing score, it will be totalled to 0.